



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199101

Roll No.

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B. Tech.

(SEM. I) (ODD SEM.) THEORY EXAMINATION, 2014-15 MATHEMATICS - I

Time : 3 Hours]

[Total Marks : 100

- Note :**
- (1) Attempt **all** questions.
 - (2) All questions carry equal marks.

1 Attempt any **two** of following :

(a) If $y = (\sinh^{-1} x)^2$, prove that

$$\left[(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + x^2y_n = 0 \right]$$

(b) If $u = x \sin^{-1} \left(\frac{x}{y} \right) + y \sin^{-1} \left(\frac{y}{x} \right)$ find the

value of $\left[x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \right]$

(c) If $u = x^2 + y^2 + z^2 - 2xyz = 1$, show that

$$\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}} = 0$$

2 Attempt any two parts of following :

(a) Trace the curve $r = a(1 + \cos \theta)$.

(b) Find the approximate value of

$$\left[(0.98)^2 + (2.01)^2 + (1.94)^2 \right]^{1/2}$$

(c) If $x = e^u \cos v$, $y = e^u \sin v$ then find

$$\frac{\partial(u, v)}{\partial(x, y)}$$

3 Attempt any two parts of following :

(a) Evaluate $\int_0^3 \int_0^{6/x} x^2 dy dx$ by changing the order

of integration.

(b) Find the area enclosed by curves $y^2 = 4ax$ and $x^2 = 4ay$.

(c) Prove that

$$\beta(m, n) = a^m b^n \int_0^{\infty} \frac{x^{m-1}}{(ax+b)^{m+n}} dx.$$

4 Attempt any two parts of following :

(a) Find ϕ if

$$\nabla \phi = \left[(y^2 - 2xyz^3) \hat{i} + (3 + 2xy - x^2z^3) \hat{j} + (6z^3 - 3x^2yz^2) \hat{k} \right]$$

(b) Evaluate by Green's theorem

$$\oint_C [(\cos x \sin y - 2xy) dx + \sin x \cos y dy]$$

where C is circle $x^2 + y^2 = 1$

(c) Verify Stoke's theorem for $\vec{F} = (y \hat{i} + z \hat{j} + x \hat{k})$

where S is upper half surface of sphere

$x^2 + y^2 + z^2 = 1$ and C is its boundary.

5 Attempt any two parts of following :

- (a) Find non-singular matrices P and Q such that P and Q is in normal form of the matrix and hence find the rank of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

- (b) Solve following system of equations :

$$2x_1 + 3kx_2 + (3k + 4)x_3 = 0$$

$$x_1 + (k + 4)x_2 + (4k + 2)x_3 = 0$$

$$x_1 + 2(k + 1)x_2 + (3k + 4)x_3 = 0$$

- (c) Find the eigen values and corresponding eigen

vectors of matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
